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THE PLANAR GRAPHS CLOSED CYCLES DETERMINATION METHOD

В.О. Вайсман, К.В. Колесникова, Д.В. Лук'янов. **Метод визначення замкнених циклів на планарних графах.** Запропоновано теоретичне обґрунтування методу аналізу топології складних організаційно-технічних систем на основі характерних властивостей матриць досяжності і її ступенів, що дозволяє автоматизувати структурний аналіз схем управління.
Ключові слова: планарні графи, системи, цикли, матриця суміжності, матриця досяжності.

В.А. Вайсман, Е.В. Колесникова, Д.В. Лукьянов. **Метод определения замкнутых циклов на планарных графах.** Предложено теоретическое обоснование метода анализа топологии сложных организационно-технических систем на основе характерных свойств матриц достижимости и ее степеней, что позволяет автоматизировать структурный анализ схем управления.
Ключевые слова: планарные графы, системы, цикли, матрица смежности, матрица достижимости.

V.A. Vaysman, E.V. Kolesnikova, D.V. Lukianov. **The planar graphs closed cycles determination method.** The theoretical justification of a method for analyzing the complex organizational-technical systems' topology is offered on the basis of reachability matrices' features and its degrees, that allows the structural analysis of management charts to be automatized.
Keywords: planar graphs, systems, cycles, adjacency matrix, reachability matrix.

The emergence of the theory of graphs associated with the work Leonhard Euler (1836), which found conditions for the existence of connected graphs cycle containing all edges without repetition [1]. This cycle is called Euler's. The next step in development of the graph theory was made by Kirchhoff (1847), which reflects the combinatorial-topological structure of the electricity network, developed an algorithm determining the maximum subgraph without cycles, called as "tree", and found it with the smallest independent system of equations of electric circuits. With the help of graphs was solved famous "traveling salesman" problem by Hamilton (1859), which was formulated as follows: "Find a simple cycle containing all vertices of a graph" [2]. Such a cycle came to be known as Hamilton's cycle.

Study of planar (flat) graphs dedicated work of many scientists. One of the most famous problems of mathematics problem — four colors also solved by using graph theory. Practical application for this problem, for example, is geographic maps coloring. Widespread use is made of methods of graph theory in the design of printed circuit boards.

Proof of the main theorem of graph theory, Euler's theorem: Graph has Euler cycle if and only if it is connected and the degree of its vertices — even, — based on evasion of vertices with the reception of those (graph edges) coloring which has already passed. This algorithm is difficult to formalize for automated problem solving.

Therefore, to solve the task of analyzing the structural scheme from the analytical definition of cycles in a complex control scheme was necessary to develop a method which, unlike the well-known theorem by Euler method cycle is determined as a result of analytical calculation, rather than heuristic search [3]. The basis for the analytical solution of the problem is the use of specific properties of the adjacency matrix.

The structural relationship between elements of the set *S* describes the adjacency matrix $[c_{ij}]_S=[i, j]$, rows and columns correspond to the tops oriented graph (digraph) structural model, and its element $c_{ij}=1$ reflects the relationship in an arc directed from vertex S_i to vertex S_j.

The value of the element $c_{ij}=1$ in the adjacency matrix indicates the connection from the top, given the line number in the top, which is marked number of the column. If $c_{ij}=0$, this corresponds to the absence of direct connection of edges from vertex *i* to vertex *j*.

The relationship between elements of sets *S* and *G*, that is, between the vertices and arcs oriented graph can be described in the form of incidence matrix $[h_{ij}]_S$, g=$[i, j]$, whose rows correspond to vertices and arcs digraph column. This h_{ij}-th element equal to +1, if S_i is the initial vertex and the arc (-1) where S_i, — the final vertex of the arc.

To describe the various structures used as the matrix of the process in which rows correspond to tops digraph and columns in specified numbers of arcs, with a plus sign — those included in the unit and with the minus sign — those emerging from the block.

Adjacency matrix, incidence and the process for the equivalent of displayed information. In practice, to describe the structural schemes often use a matrix process as a more compact shape description. A structure used for the analysis of adjacency matrix, which has specific properties [3]. If it consistently reduced to levels 2, 3, ..., *n*, then the element c_{ij} *n*-th degree adjacency matrix shows the relationship between the *i*-th and *j*-th vertices in *n* arcs. With the rise to the *n*-th degree of adjacency matrix and the presence of a feedback loop some diagonal elements are different from zero, which means the relationship of the *i*-th in the *i*-in the top graph. This connection can only be in a closed circuit. In matrix multiplication applies the usual rule, under which

$$c_{ij}^{n+1} = \sum_{k=1}^{m} c_{ik}^{n} c_{kj} ,$$

where $n = 1, 2, \ldots, m-1$;

m — total number of vertices in the circuit.

To analyze the structural models has value only difference element matrix $[c_{ij}]$ from scratch, so the formation of this matrix is applied Boolean algebra

0+0=0	0×0=0
0+1=1	0×1=0
1+1=1	1×1=1.

In the published work on the structural analysis of circuits driven, often without evidence, the recommendations in the form of "recipes" on how to provide closed contours [1, 2]. It is therefore necessary to justify theoretically the accuracy of the proposed solutions.

The proof of the theorem on determining all the ways of the graph. The theoretical justification of methods of management structures are very important in project management, because the structure of production systems that aim to implement projects and programs, and information links in these project-driven organizations significantly affect performance. Keep the research methods of presentation of vari-

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ous structures using adjacency matrix, we consider the properties of the adjacency matrix and its stages in the application of these properties for the analysis of structural schemes for edge control.

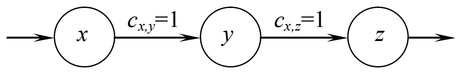


Fig. 1. Fragment oriented graph

Lemma 1. In the adjacency matrix of a linear subgraph arcs that enter and leave with a top, are two elements that are shifted from the main diagonal of a column in the direction of arcs, if and only if the three peaks oriented graph presents the adjacent columns.

Proof. By rule display graphs in adjacency matrix row numbers correspond to number peaks, from which comes the arc. A column numbers — top number, which includes the arc. Note that in any digraph with a loop, here is the linear part of the path in the direction of arcs digraph and feedback (arc), which forms a loop.

As the numbers of vertices digraph play more role identifiers peaks and determine the binding order of the adjacency matrix, and does not affect the structure of links between nodes, accept the assumption that the digraph vertices can be numbered arbitrarily. Therefore impose a condition on the numbering tops digraph: the linear subgraph $g\in \mathbf{G}$ peaks are numbered in the direction of arcs digraph (fig. 1).

Therefore, we can take such numbers: $x=i$; $y=i+1$; $z=i+2$. In this case, the adjacency matrix of rows and columns corresponding to the tops x, y, z, will be placed in sequence, then

$$c_{i,i+1}=c_{x,y}=1;$$

$$c_{i+1,i+2}=c_{y,z}=1.$$

Elements of adjacency matrix $c_{i,i+1}=1$ and $c_{i+1,i+2}=1$ a shifted one column from the main diagonal. Arches linear part of the closed loop digraph adjacency matrix are displayed in parallel to the main diagonal, shifted from it to a column if the peaks are located in the adjacency matrix sequentially in the direction of arcs digraph.

Arc, which does not form a diagonal, not related to the linear part of the arc digraph. This conclusion is the use of Lemma 1 for the case of a sequence of vertices that are connected by arcs in the path.

Column i contains only zero elements, since no arc not included in the peak i. Line $i+4$ also consist of zero, indicating no direct connection from this to other nodes. Path or cycle on the graph includes vertices $i+1$, $i+2$ and $i+3$, are connected by sequence three arcs: $g_{i+1,i+2}=1$; $g_{i+2,i+3}=1$; $g_{i+3,i+1}=1$.

Lemma 2. Elements of all columns circuit, except the last, adjacency matrix of degree are shifted in degree n +1 in a column in the direction of edges digraph.

Proof. We use the property for arbitrary numbers of vertices (see Lemma 1). This nonzero elements of adjacency matrix of degree n = 1

$$c_{i,i+1}=1,\ i=k,\ k+1,\ \ldots,\ m;\ k\in 1,\ \ldots,\ m-1;$$

$$c_{m,k}=1,$$

where k, m — start and end vertices belonging to the contour, $k<m$.

Elements of adjacency matrix of degree n +1 are determined by the formula:

$$c_{ij}^{n+1}=\sum_{h=k}^{m}c_{ih}^{n}c_{hj},\ j=1,\ 2,\ldots,m;\ i=1,\ 2,\ldots,m.$$

Consider the formation of a string s adjacency matrix of degree n+1.

$$c_{s,1}^{n+1}=c_{s,1}^{n}c_{1,1}+c_{s,2}^{n}c_{2,1}+c_{s,3}^{n}c_{3,1}+\cdots+c_{s,m}^{n}c_{m,1};$$

$$c_{s,2}^{n+1}=c_{s,1}^{n}(c_{1,2})+c_{s,2}^{n}c_{2,2}+c_{s,3}^{n}c_{3,2}+\cdots+c_{s,m}^{n}c_{m,2};$$

$$c_{s,3}^{n+1}=c_{s,1}^{n}c_{1,3}+c_{s,2}^{n}(c_{2,3})+c_{s,3}^{n}c_{3,3}+\cdots+c_{s,m}^{n}c_{m,3};$$

$$\vdots$$

$$c_{s,m}^{n+1}=c_{s,1}^{n}c_{1,m}+c_{s,2}^{n}c_{2,m}+\cdots+c_{s,m-1}^{n}(c_{m-1,m})+c_{s,m}^{n}c_{m,m},$$

where m — item number in the row.

Brackets allocated non-zero elements $c_{i,i+1}=1(i=1,\ 2,\ \ldots,\ m-1)$ adjacency matrix. Rejecting the remaining elements, and elements taking values in parentheses $c_{i,i+1}=1$ $(i=1,\ 2,\ \ldots,\ m-1)$, we obtain in the general case:

$$c_{s,h}^{n+1}=c_{s,h-1}^{n};\ h=2,\ 3,\ldots,m.$$

Lemma 3. Items in the last column contour adjacency matrix of degree n in the transition to a pass degree $n+1$ in the first column of the path.

Proof. Let the given subgraph of the circuit shown adjacency matrix with the conditions adopted in Lemma 2. Consider forming a column k subgraph adjacency matrix of degree n+1. The elements of column k are calculated according to the rules of multiplication of matrices:

$$c_{k,k}^{n+1}=c_{k,k}^{n}c_{k,k}+[c_{k,k+1}^{n}]\big|_{n=1}c_{k+1,k}+[c_{k,k+2}^{n}]\big|_{n=2}c_{k+2,k}+\cdots+c_{k,m-1}^{n}c_{m-1,k}+c_{k,m}^{n}(c_{m,k});$$

$$c_{k+1,k}^{n+1}=c_{k+1,k}^{n}c_{k,k}+c_{k+1,k+1}^{n}c_{k+1,k}+[c_{k+1,k+2}^{n}]\big|_{n=1}c_{k+2,k}+\cdots+c_{k+1,m-1}^{n}c_{m-1,k}+c_{k+1,m}^{n}(c_{m,k});$$

$$c_{k+2,k}^{n+1}=c_{k+2,k}^{n}c_{k,k}+c_{k+2,k+1}^{n}c_{k+1,k}+c_{k+2,k+2}^{n}c_{k+2,k}+\cdots+c_{k+2,m-1}^{n}c_{m-1,k}+c_{k+2,m}^{n}(c_{m,k});$$

$$\vdots$$

$$c_{m-1,k}^{n+1}=c_{m-1,k}^{n}c_{k,k}+c_{m-1,k+1}^{n}c_{k+1,k}+c_{m-1,k+2}^{n}c_{k+2,k}+\cdots+c_{m-1,m-1}^{n}c_{m-1,k}+[c_{m-1,m}^{n}]\big|_{n=1}(c_{m,k});$$

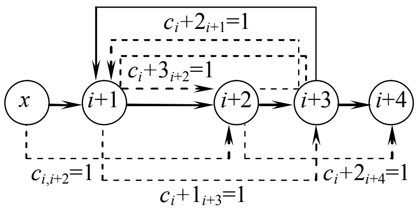
$$c_{m,k}^{n+1}=c_{m,k}^{n}c_{k,k}+c_{m,k+1}^{n}c_{k+1,k}+c_{m,k+2}^{n}c_{k+2,k}+\cdots+c_{m,m-1}^{n}c_{m,k}+c_{m,m}^{n}(c_{m,k}).$$

Brackets allocated non-zero elements $c_{i,i+1}=1$ $(i=k,\ k+1,\ \ldots,\ m;\ k\in 1,\ \ldots,\ m-1)$ adjacency matrix of the first degree. Rejecting the null elements and elements taking values in parentheses $c_{m,k}=1$, we obtain:

$$c_{s,k}^{n+1}=c_{s,m}^{n};\ s=k,\ k+1,\ldots,m.$$

Lemma 4. Degrees of adjacency matrix from 1 to n represent links between nodes of the graph, respectively, in 1...n arcs.

Proof. As defined in Lemma 2 elements of the column circuit, except the last, adjacency matrix of degree n are shifted in degree $n+1$ in a column in the direction of edges digraph. That is, every $n+1$ degree reflects the relationship of the i-th to $n+1$ vertices. Relationships are derived from the 2-degree adjacency matrix display links in the column through a transit peak (dotted line, fig. 2, *a*).



a

Second degree		In the top				
		i	*i*+1	*i*+2	*i*+3	*i*+4
From the top	*i*	0	0	1	0	0
	i+1	0	0	0	1	0
	i+2	0	1	0	0	1
	i+3	0	0	1	0	0
	i+4	0	0	0	0	0

b

Fig. 2. Display links in the adjacency matrix of the 2nd degree

Apparently, the new links connecting those vertices that are in the initial matrix are connected by two arcs. These conclusions are also true for the 3rd degree adjacency matrix, with the difference that has revealed ties passing through three arches and two vertices.

Theorem. Boolean sum of adjacency matrices of degrees from 1 to m — matrix range, forming a graph of all the ways of schemes, including closed circuit.

Proof. We use the conclusions of Lemma 4. For the matrix **R** of all the ways digraph reach ability or matrix forms a Boolean sum of all degrees of adjacency matrix $\mathbf{C}^n$, $n=1,\ 2,\ \ldots,\ m$, where m — the total number of vertex oriented graph. The elements r_{ij} matrix defined range of conditions:

$$r_{ij} = \begin{cases} 1, \text{ if } \ (c_{ij}^{n-1} = 1) \vee (c_{ij}^{n} = 1) \\ 0, \text{ if } \ (c_{ij}^{n-1} = 0) \wedge (c_{ij}^{n} = 0) \end{cases}.$$

The resulting matrix contains all the range $\mathbf{R}^n$ connections from vertices i to vertex j in n arcs (fig. 3). As the degrees of adjacency matrices that are sorrowful, are filled units reach the matrix elements according to Lemma 2 on successive elements in the displacement direction of arcs digraph. The completed unit of square submatrices (4×4) shows that all its vertices are related in the direction of arcs. And this is the description of all possible ways to digraph.

R^2

0	1	1	0	0
0	0	1	1	0
0	1	0	1	1
0	1	1	0	1
0	0	0	0	0

R^3

0	1	1	1	0
0	1	1	1	1
0	1	1	1	1
0	1	1	1	1
0	0	0	0	0

R^4

0	1	1	1	1
0	1	1	1	1
0	1	1	1	1
0	1	1	1	1
0	0	0	0	0

Fig. 3. Matrix range

An indication of the existence of a path in the circuit is different from zero main diagonal elements of the matrix $\mathbf{R}^n$. In the matrix $\mathbf{R}^3$ has shown edge (fig. 3).

To select all subsystems that are reflected in the planar graph and covered feedback, use the method replace all directions arcs oriented graph of inverse transpose of the matrix range $\mathbf{R} \Rightarrow \mathbf{R}^{\mathbf{T}}$, followed by superposition. The elements of the matrix superposition $\mathbf{W}=\mathbf{R} \bigcap \mathbf{R}^{\mathbf{T}}$ is formed as follows:

$$w_{ij} = \begin{cases} 1, \text{ if } \ (r_{ij} = 1) \wedge (r_{ij}^{\mathrm{T}} = 1); \\ 0, \text{ if } \ (r_{ij} = 0) \vee (r_{ij}^{\mathrm{T}} = 0). \end{cases}$$

Nonzero elements of the main diagonal of the matrix W indicate a string that containsall vertices of a closed circuit. Edge allows improving the management structures. This informative is not only the end result — the matrix superposition $\mathbf{W}^n$, and the results that show the formation of the control circuit.

The theorem is the basis for solving problems of complex analysis of the topology of organizational and technical systems with the analytical definition of cycles. The proposed method unlike known by Euler's theorem to determine the cycles on planar graphs as a result of the calculation, rather than heuristic search.

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